Sample design problem

The "final" equation in the course for the determination of dose from a point source that includes attenuation and buildup is given by:

$$D_{tot} = D_{unc} B(E_0, \mu t)$$

where:

$$D_{unc} = \frac{S_p \hat{R}}{4\pi r^2} e^{-\mu t}$$

and

$$B(E_0, \mu t) = A_1 e^{-\alpha_1 \mu t}$$

Putting this all together, and combining terms for convenience, gives:

$$D_{tot} = \frac{[A_1 S] R}{4\pi r^2} e^{-(1+\alpha_1)\mu t}$$

The ways that each of these terms is determined from data given in the text are shown in the following example. To put numbers on it, we assume you want to find the maximum surface dose on top of an 80 cm slab of concrete that covers a 2 curie point source of 5 MeV photons.

- S_p is found from the decay rate times the number of gammas (and their energy) released per decay.

    For the example, this would be:

$$\begin{aligned} S_p &= (2\ \text{Ci})\left(3.7\times 10^{10}\ \text{disintegrations/Ci}\right)(1\ \text{photon/disintegration}) \\ &= 7.4\times 10^{10}\ \text{photons/sec} \end{aligned}$$

For cases in which I give you the mass of an isotope, you have to find the activity yourself, using Appendix H data.

    Example: Source is 1 milligram of ^{46}Sc:

- $\hat{R}$ is found from Appendix D of the text. If not stated, the most conservative to use is the Anteroposterior (AP) value by energy

    For the example, this would be $\hat{R} = 14.7\times 10^{-12}\ \text{Sv-cm}^2$

    I often have you work in mrem/hr, so this is a good place to get in those units:

$$\hat{R} = \left(14.7 \times 10^{-12} \text{ Sv-cm}^2\right) \cdot \frac{10^5 \text{ mrem}}{Sv} \cdot \frac{3600 \text{ sec}}{\text{hr}}$$
$$= 5.292 \times 10^{-3} \text{ mrem-cm}^2\text{-sec/hr}$$

- *r* is the distance from source to detector. For the example, this is 80 cm

- μ is the linear attenuation coefficient. This is found from Appendix C values of m/r multiplied by the density of the material (found in the footnotes from the Appendix C table used, although you are expected to know that water density is 1 g/cc)

  Example: Attenuating material is water, so 5 MeV values for concrete are used:

$$\mu = \frac{\mu}{\rho}\rho = \left(2.895 \times 10^{-2} \text{ cm}^2/\text{g}\right)\left(2.3 \text{ g/cm}^3\right)$$
$$=0.06659 \text{ cm}^{-1}$$

- *t* is the thickness of the shielding material, which is sometimes equal to *r* and sometimes not. For the example, this is 80 cm

- A_1 and α_1 are found from the single-term form of the Taylor form for buildup. This table is in Chapter 7. For the example, these values are 1.8 and 0.14, respectively.

- E_0 is the initial energy of the photon. For the example, this is 5 MeV.

- τ is the optical thickness of the shield, or the number of mean free paths that the particle has to travel through the shield.
  Example: Attenuating material is water, so 5 MeV values for concrete are used:

$$\tau = \mu t = \left(0.06685 \text{ cm}^{-1}\right)\left(80 \text{ cm}\right)$$
$$=5.327$$

$B(E_0,\tau)$ is found from the equation given above. For the example, this is:

$$B(E_0,\tau) = A_1 e^{-\alpha_1 \tau} = 1.8e^{-(-0.14)(5.327)} = 3.794$$

(The negative-negative in the exponential is a little tricky; always remember that buildup is buildUP, and should always be > 1.)

<u>Solving the problem</u>
Putting this all together (using the example) would give us:

$$
\begin{aligned}
D_{tot} &= \frac{[A_1 S] R}{4\pi r^2} e^{-(1+\alpha_1)\mu t} \\
&= \frac{(1.8)\left(7.4\times 10^{10}\,\text{photons/sec}\right)\left(5.292\times 10^{-3}\ \text{mrem-cm}^2\text{-sec/hr}\right)}{4\pi\,(80)^2} e^{-(.86)\left(0.06659\ \text{cm}^{-1}\right)(80\,\text{cm})} \\
&= 89.755\ \text{mrem/hr}
\end{aligned}
$$

Design problem

I will often give you a design problem, usually involving coverage of a source with a layer of water, concrete, etc. So, $t=r$.

To show how this is done, assume I asked for the water coverage that would reduce the above dose rate to 0.5 mrem/hr at the surface.

Returning to the above equation, we would have:

$$
D_{tot} = \frac{[A_1 S] R}{4\pi r^2} e^{-(1+\alpha_1)\mu t}
$$

$$
0.5\ \text{mrem/hr} = \frac{(1.8)\left(7.4\times 10^{10}\,\text{photons/sec}\right)\left(5.292\times 10^{-3}\ \text{mrem-cm}^2\text{-sec/hr}\right)}{4\pi r^2} e^{-(.86)\left(0.06659\ \text{cm}^{-1}\right)r}
$$

$$
= \frac{5.609\times 10^{7}}{r^2} e^{-\left(0.05726\ \text{cm}^{-1}\right)r}\ \ \text{mrem/hr}
$$

or

$$
\frac{0.5}{5.609\times 10^{7}} = \frac{e^{-0.05726r}}{r^2}
$$

or

$$
\frac{e^{-0.05726r}}{r^2} = 8.914\times 10^{-9}
$$

If you are lucky enough to have EXCEL open, using the "goal seek" option quickly delivers the answer as r=148.92 cm. Solving by trial-and-error might be a little more tedious.

A recurrence relation to solve for the r in the exponential (converges in a few iterations):

$$
\begin{aligned}
e^{-0.05726r} &= 8.914\times 10^{-9} r^2 \\
r &= \frac{\ln\left(8.914\times 10^{-9} r^2\right)}{-0.05726} = \frac{\ln\left(8.914\times 10^{-9}\right) + 2\ln(r)}{-0.05726} \\
&= \frac{\ln\left(8.914\times 10^{-9}\right)}{-0.05726} + \frac{2\ln(r)}{-0.05726} \\
r_{new} &= 323.69 - 34.927\ln(r_{old})
\end{aligned}
$$