NE 582 Final

Takehome due midnight May 1, 2019

1. If we simplify the "I shot an arrow into the air" problem to:

$$\frac{d^2 z(t)}{dt^2} = -g(t),\ z(0) = 0,\ z'(0) = 100,\ \text{g(t)=10 for all t}$$

   then the arrow (or whatever) will reach its peak height in 8 seconds. Write and solve a linked Monte Carlo problem to find the maximum height z(10) (comparing it to the analytical solution, which I assume you can find). Do not do ANY integrals. (I even, stupidly, want you to sample g(t) even though it is a constant.)

   a. Give me the general MC algorithm.
   b. Give me the specific algorithm where both times are chosen uniformly between 0 and 10 seconds.
   c. Code it and include a code listing.
   d. Run it and show me your results (with standard deviation). Compare to the analytic result.

2. You are reading a technical paper and it says that they "Sample x between 0 and 1 from a beta distribution with alpha=2 and beta=5." But they do not give the details.

   You find from your statistics book that the unnormalized beta distribution is given by:

$$\pi(x) \sim x^{\alpha-1}(1-x)^{\beta-1}$$

   with an average value of

$$\bar{x} = \frac{\alpha}{\alpha+\beta}$$

   a. Develop a rejection algorithm for choosing x for alpha=2 and beta=5.
   b. Test it by binning with 100 divisions between 0 and 100.
   c. Test its ability to reproduce the mean.

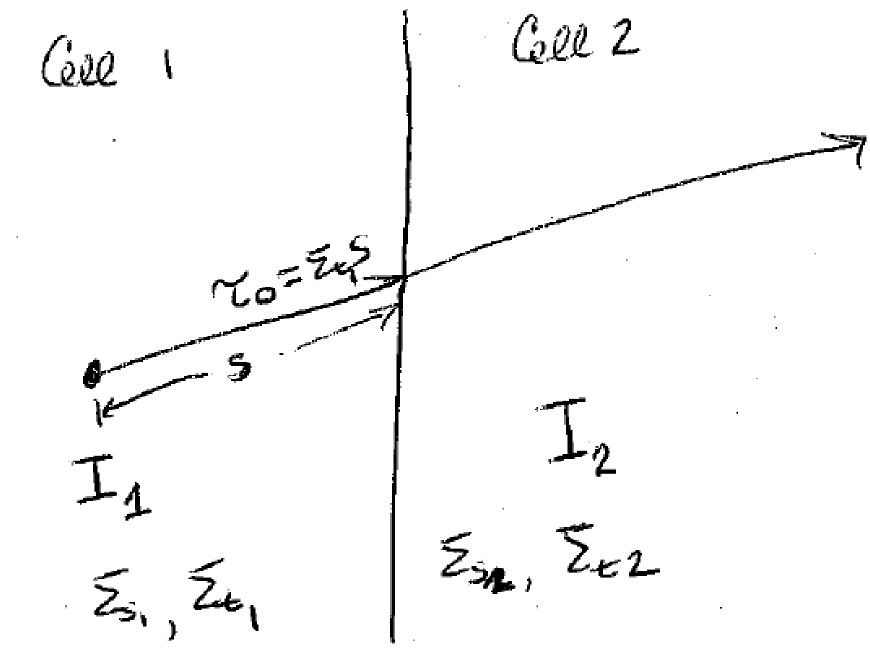
3. Starting with the infinite medium integral transport equation:

$$\phi(E) = \frac{S(E)}{\sigma_t(E)} + \int_0^\infty \frac{\sigma_s(E' \to E)}{\sigma_t(E)} \phi(E')\,dE'$$

   develop a Monte Carlo algorithm to sample the flux as a function of energy (i.e., the formulas for w_0 and w_{i+1} (given w_i)). Leave it as a general algorithm (i.e., with the PDFs still in the weight equation.)

4. Based on our class's discussion of the forced collision method, you think of a nifty improvement of the cell weighting procedure: When a particle has been born or emerged from a scattering collision with known direction and energy (therefore known total cross sections), you use the relative importances of the cell it is in and the cell it is next heading for to optimally bias the probability that it reaches the next cell (versus colliding in the current cell).

   You sketch your idea like this (assuming one energy group):

   where the I_1 is the expected contribution of a particle released in Cell 1 and I_2 is the expected contribution if it escapes.

   Normally, of course, the probability that the particle would reach Cell 2 would be

$$\pi_{escape} = e^{-\Sigma_{t1} s}$$

   But you propose to use a different escape probability to completely eliminate the variance contribution of this decision. What would it be? What would the weight correction be if it escapes? What would the weight correction be if it does not escape?